THE APRIL MEETING IN NEW YORK

The three hundred eighty-sixth meeting of the American Mathematical Society was held at Columbia University on Friday and Saturday, April 3–4, 1942. The attendance was approximately two hundred seventy-five including the following two hundred four members of the Society:

Milton Abramowitz, C. R. Adams, Felix Adler, Leonidas Alaoglu, C. B. Allendoerfer, Lawrence Annenberg, R. G. Archibald, B. H. Arnold, L. A. Aroian, Y. K. Bal, J. D. Bankier, J. J. Barron, Alfred Basch, P. T. Bateman, M. F. Becker, P. O. Bell, Stefan Bergman, Garrett Birkhoff, Gertrude Blanch, M. L. Boas, R. P. Boas, H. F. Bohnenblust, Samuel Borofsky, C. C. Bramble, Richard Brauer, Jewell H. Bushey, J. H. Bushey, W. E. Byrne, W. B. Campbell, Claude Chevalley, D. E. Christie, Alonzo Church, M.D. Clement, Harvey Cohn, J. B. Coleman, L. M. Comer, T. F. Cope, Richard Courant, W. H. H. Cowles, H. B. Curry, M. D. Darkow, Norman Davids, A. H. Diamond, R. P. Dilworth, J. L. Doob, Jesse Douglas, C. H. Dowker, Jacques Dutka, John Dyer-Bennet, J. E. Eaton, W. F. Eberlein, M. L. Elveback, Benjamin Epstein, Paul Erdös, J. M. Feld, W. K. Feller, Aaron Fialkow, F. A. Ficken, W. B. Fite, R. M. Foster, G. A. Foyle, J. S. Frame, K. O. Friedrichs, Orrin Frink, Guido Fubini, R. E. Fullerton, H. P. Geiringer, J. H. Giese, B. P. Gill, A. M. Gleason, M. C. Gray, H. J. Greenberg, Lewis Greenwald, C. C. Grove, Laura Guggenbühl, N. G. Gunderson, Jacques Hadamard, Theodore Hailperin, F. C. Hall, P. R. Halmos, K. E. Hazard, M. H. Heins, Edward Helly, Olaf Helmer, E. H. C. Hildebrandt, Banesh Hoffmann, T. R. Hollcroft, Harold Hotelling, E. M. Hull, Witold Hurewicz, W. A. Hurwitz, L. C. Hutchinson, R. P. Isaacs, Nathan Jacobson, R. A. Johnson, G. K. Kalisch, Edward Kasner, S. A. Kiss, S. C. Kleene, R. B. Kleinschmidt, J. R. Kline, E. R. Kolchin, B. O. Koopman, Arthur Korn, M. E. Ladue, J. S. Le Caine, Solomon Lefschetz, B. A. Lengyel, Howard Levi, Norman Levinson, E. R. Lorch, A. N. Lowan, N. H. McCoy, J. C. C. McKinsey, Brockway McMillan, L. A. MacColl, J. K. L. MacDonald, C. C. MacDuffee, G. W. Mackey, G. R. MacLane, H. M. MacNeille, M. S. Macphail, P. T. Maker, Imanuel Marx, W. P. Mason, A. E. Meder, H. L. Mintzer, R. E. von Mises, E. C. Molina, Deane Montgomery, Richard Morris, Marston Morse, G. W. Mullins, F. J. Murray, C. A. Nelson, N. D. Nelson, Philip Newman, K. L. Nielsen, C. O. Oakley, Rufus Oldenburger, Oystein Ore, J. C. Oxtoby, N. G. Parke, L. G. Peck, A. S. Peters, George Polya, E. L. Post, Willy Prager, M. H. Protter, R. G. Putnam, J. F. Randolph, C. J. Rees, M. S. Rees, Eric Reissner, R. W. Rempfer, H. J. Riblet, Moses Richardson, R. G. D. Richardson, J. F. Ritt, H. E. Robbins, J. H. Roberts, M. S. Robertson, S. L. Robinson, Benjamin Rosenbaum, J. E. Rosenthal, Raphaël Salem, Hans Samelson, Arthur Sard, L. J. Savage, Rubin Schatten, M. G. Scherberg, I. J. Schoenberg, Alberta Schuettler, I. E. Segal, I. M. Sheffer, Seymour Sherman, C. A. Shook, P. A. Smith, D. C. Spencer, Wolfgang Sternberg, J. J. Stoker, R. E. Street, R. G. Sturm, Alvin Sugar, J. L. Synge, Otto Szász, J. D. Tamarkin, M. E. Taylor, J. W. Tukey, J. L. Vanderslice, G. L. Walker, Henry Wallman, R. M. Walter, W. R. Wasow, Louis Weisner, Hermann Weyl, A. L. Whiteman, D. V. Widder, V. A. Widder, Norbert Wiener, John Williamson, Audrey Wishard, Oscar Zariski, Leo Zippin.

The meeting opened Friday morning with a section for papers in

analysis and applied mathematics, Professor Norman Levinson presiding, and another for algebra, geometry and logic, Professor Alonzo Church presiding.

Friday afternoon there was a symposium on topics in the theory of functions, at which Vice President C. C. MacDuffee and President Marston Morse presided. These addresses were given: *On entire functions of exponential type*, by Dr. R. P. Boas of Duke University; and, *On the zeros of the derivatives of a function and its analytic character*, by Professor George Polya of Smith College. The discussion was led by Professor Norman Levinson of the Massachusetts Institute of Technology and Professor I. J. Schoenberg of the University of Pennsylvania.

Saturday morning was devoted to a symposium on aeronautics at which Vice President J. D. Tamarkin presided. There was an address on *Theory of wing flutter*, by Dr. Theodore Theodorsen of Langley Field, Virginia, and on *New approach to the three-dimensional wing theory*, by Dr. R. E. von Mises of Harvard University. The discussion was led by Professor Willy Prager of Brown University, Dr. Eric Reissner of the Massachusetts Institute of Technology, Professor K. O. Friedrichs of New York University, and Professor J. L. Synge of the University of Toronto.

Titles and cross references to the abstracts of papers read follow below. Papers whose abstract numbers are followed by the letter *t* were read by title. Papers numbered 1–8 were read before the section for analysis and applied mathematics; those numbered 9–17 before the section for algebra, geometry and logic; those numbered 18–39 were read by title. Paper 7 was read by Professor Roberts, and paper 12 by Mr. Tuan. Dr. Bers was introduced by Dr. A. M. Gelbart.

1. Lipman Bers: *A property of harmonic functions.* (Abstract 48-5-170.)

2. Otto Szász: *On sequences of polynomials and the distribution of their zeros.* (Abstract 48-5-193.)

3. J. H. Bushey: *The distribution function of the sample total under the type β hypothesis.* (Abstract 48-5-217.)

4. Hermann Weyl: *Solution of the simplest boundary-layer problems in hydrodynamics.* (Abstract 48-5-198.)

5. Hilda P. Geiringer: *On the numerical solution of linear problems by group iteration.* (Abstract 48-5-202.)

6. Stefan Bergman: *Three-dimensional flow of a perfect incompressible fluid and its singularities.* (Abstract 48-5-201.)

7. J. H. Roberts and Paul Civin: *Sections of continuous collections.* (Abstract 48-5-224.)

8. Witold Hurewicz: *Ergodic theorem without invariant measure.* (Abstract 48-7-235.)

9. B. H. Arnold: *Rings of transformations of certain vector spaces.* Preliminary report. (Abstract 48-5-142.)

10. F. A. Ficken: *On Rosser's generalization of Euclid's algorithm.* Preliminary report. (Abstract 48-5-151.)

11. Rufus Oldenburger: *The index of a quadratic form for an arbitrary field.* (Abstract 48-5-162.)

12. Richard Brauer and Hsio-Fu Tuan: *Some remarks on simple groups of finite order.* Preliminary report. (Abstract 48-5-147.)

13. J. S. Frame: *Double coset matrices and group characters.* (Abstract 48-5-152.)

14. S. C. Kleene: *On the forms of the predicates in the theory of constructive ordinals.* Preliminary report. (Abstract 48-5-215.)

15. L. C. Hutchinson: *On the linear line complex in n-space.* (Abstract 48-5-208.)

16. Aaron Fialkow: *Conformal differential geometry of a subspace.* (Abstract 48-5-207.)

17. R. P. Dilworth: *The partition lattice.* (Abstract 48-5-150.)

18. Stefan Bergman: *Determination of pressure in the two-dimensional flow of an incompressible perfect fluid.* (Abstract 48-5-200-*t*.)

19. J. H. Bushey: *The distribution function of the mean under the type α hypothesis.* (Abstract 48-5-216-*t*.)

20. J. H. Bushey: *The products of certain discrete and continuous orthogonal polynomials.* (Abstract 48-5-218-*t*.)

21. Leonard Carlitz: *On the reciprocal of certain types of Hurwitz series.* (Abstract 48-5-148-*t*.)

22. F. G. Dressel: *The fundamental solution of the parabolic equation.* II. (Abstract 48-5-171-*t*.)

23. A. M. Gelbart: *Bounds for pressure in a two-dimensional flow of an incompressible perfect fluid.* (Abstract 48-5-203-*t*.)

24. P. R. Halmos and Hans Samelson: *On monothetic groups.* (Abstract 48-7-250-*t*.)

25. Nathan Jacobson: *Classes of restricted Lie algebras of characteristic p.* II. (Abstract 48-7-231-*t*.)

26. Meyer Karlin: *Characterization of the extremals of a variation problem of higher order in the plane.* Preliminary report. (Abstract 48-5-175-*t*.)

27. Joseph Lehner: *On certain irrational modular equations.* (Abstract 48-5-156-*t*.)

28. F. A. Lewis: *Generators of permutation groups simply isomorphic with $LF(2, p^n)$.* (Abstract 48-5-157-*t*.)

29. A. N. Lowan, H. E. Salzer, and Abraham Hillman: *Coefficients of differences in the expansion of derivatives in terms of advancing differences.* (Abstract 48-5-176-*t*.)

30. Rufus Oldenburger: *Repeated linear factors of forms.* (Abstract 48-5-161-*t*.)

31. Harry Pollard: *An inversion formula for the Stieltjes transform.* (Abstract 48-5-182-*t*.)

32. Harry Pollard: *On subseries of a convergent series.* (Abstract 48-5-183-*t*.)

33. H. A. Rademacher: *On the Bloch-Landau constant.* (Abstract 48-5-185-*t*.)

34. M. F. Smiley: *A remark on S. Kakutani's characterization of* (L)*-spaces.* (Abstract 48-5-191-*t*.)

35. M. F. Smiley and W. R. Transue: *Applications of transitivities of betweenness in lattice theory.* (Abstract 48-5-165-*t*.)

36. J. L. Walsh: *On the overconvergence, degree of convergence, and zeros of sequences of analytic functions.* (Abstract 48-5-196-*t*.)

37. G. T. Whyburn: *Coherent and saturated collections.* (Abstract 48-5-226-*t*.)

38. G. T. Whyburn: *On the interiority of mappings.* (Abstract 48-5-227-*t*.)

39. G. T. Whyburn: *Unitary subcontinua.* (Abstract 48-5-228-*t*.)

T. R. HOLLCROFT,
Associate Secretary

THE APRIL MEETING IN BERKELEY

The three hundred eighty-seventh meeting of the American Mathematical Society was held at the University of California, Berkeley, on Saturday, April 11, 1942. The attendance was about seventy, including the following thirty-six members of the Society:

S. P. Avann, H. M. Bacon, E. W. Barankin, K. E. Benson, B. A. Bernstein, H. F. Blichfeldt, T. C. Doyle, G. C. Evans, A. L. Foster, D. L. Fuller, S. M. Hallam, M. A. Heaslet, Einar Hille, Alfred Horn, R. C. James, D. H. Lehmer, A. O. Leuschner, S. H. Levy, Hans Lewy, W. R. Mann, W. A. Manning, A. D. Michal, E. M. Morenus, C. B. Morrey, Jr., A. P. Morse, A. R. Noble, R. M. Robinson, E. B. Roessler, P. C. Rosenbloom, A. C. Schaeffer, W. H. Simons, Gabor Szegö, S. P. Timoshenko, F. A. Valentine, W. M. Whyburn, A. R. Williams.

The meeting opened in the morning with a general session for contributed papers at which Professor W. M. Whyburn presided. By invitation of the Program Committee, Professor W. A. Manning of Stanford University delivered an hour address on *The simply transitive permutation groups*. Professor B. A. Bernstein presided at this lecture.

The afternoon session was devoted to a symposium on applied mathematics, Professor Gabor Szegö presiding. Professor Theodore von Kármán spoke on *Some problems in dynamics of compressible fluids*; Professor E. E. Weibel on *The photo-elastic method of stress determination*; and Professor C. F. Richter on *Mathematical questions in seismology*.

Titles and cross references to the abstracts of the papers read follow below. Papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1–5 were read at the morning session; those numbered 6–14 were read by title.

1. G. E. Forsythe and A. C. Schaeffer: *A remark on Toeplitz matrices*. (Abstract 48-3-108.)

2. F. A. Valentine: *On the extension of a vector function so as to preserve a Lipschitz condition*. (Abstract 48-5-195.)

3. R. C. James: *Normability of topological abelian groups*. (Abstract 48-5-221.)

4. S. P. Avann: *On complete classes of projective quotients in a modular lattice*. (Abstract 48-5-145.)

5. Einar Hille: *Notes on linear transformations*. IV. *Representation of semi-groups*. (Abstract 48-7-234.)

6. W. A. Mersman: *Heat conduction in an infinite composite solid with an interface resistance*. (Abstract 48-5-205-*t*.)

7. Walter Leighton and W. J. Thron: *On value regions of continued fractions.* (Abstract 48-3-113-*t*.)

8. R. A. Beaumont: *Projections of the prime-power abelian group of order p^m and type $(m-1, 1)$.* (Abstract 48-3-103-*t*.)

9. W. A. Mersman: *Heat conduction in a finite composite solid.* (Abstract 48-5-204-*t*.)

10. Knox Millsaps: *Abstract polynomials in non-abelian groups.* (Abstract 48-5-158-*t*.)

11. S. P. Avann: *A numerical condition for modularity of a finite lattice.* (Abstract 48-5-144-*t*.)

12. E. J. Pinney: *Calculus of variations in abstract spaces and related topics.* I. (Abstract 48-5-180-*t*.)

13. E. J. Pinney: *Calculus of variations in abstract spaces and related topics.* II. (Abstract 48-5-181-*t*.)

14. A. D. Michal: *A theory of fluctuations in Riemannian spaces.* Preliminary report. (Abstract 48-5-209-*t*.)

A. D. MICHAL,
Acting Associate Secretary

THE APRIL MEETING IN CHICAGO

The three hundred eighty-eighth meeting of the American Mathematical Society was held at the University of Chicago, Chicago, Illinois, on Friday and Saturday, April 17–18, 1942.

The attendance was one hundred eighty-three including the following one hundred sixty-one members of the Society:

A. A. Albert, J. V. Atanasoff, W. L. Ayres, Reinhold Baer, R. M. Ballard, R. H. Bardell, Walter Bartky, E. F. Beckenbach, J. C. Bell, D. L. Bernstein, S. F. Bibb, E. E. Blanche, G. A. Bliss, Henry Blumberg, L. M. Blumenthal, R. W. Brink, E. L. Buell, H. E. Burns, I. W. Burr, L. E. Bush, Herbert Busemann, Albert Cahn, W. C. Carter, E. W. Chittenden, L. W. Cohen, B. H. Colvin, J. J. Corliss, L. L. Cronvich, A. R. Craw, E. L. Crow, D. R. Curtiss, W. M. Davis, M. M. Day, L. E. Dickson, J. M. Dobbie, Roy Dubisch, W. D. Duthie, Phillip Eddy, Samuel Eilenberg, R. S. Embree, W. S. Erickson, E. B. Escott, H. P. Evans, H. S. Everett, G. M. Ewing, J. V. Finch, L. R. Ford, R. H. Fox, Bernard Friedman, H. L. Garabedian, B. H. Gere, H. A. Giddings, B. E. Gillam, J. W. Givens, E. L. Godfrey, R. A. Good, G. D. Gore, L. M. Graves, L. W. Griffiths, V. G. Grove, G. J. Haltiner, R. W. Hamming, O. G. Harrold, W. L. Hart, M. L. Hartung, G. E. Hay, C. T. Hazard, A. E. Heins, E. D. Hellinger, M. R. Hestenes, J. F. Heyda, T. H. Hildebrandt, A. S. Householder, H. K. Hughes, D. H. Hyers, M. H. Ingraham, Dunham Jackson, R. D. James, R. N. Johanson, Fritz John, R. E. Johnson, G. K. Kalisch, Samuel Kaplan, Wilfred Kaplan, William Karush, D. E. Kibbey, Lois Kiefer, J. R. Kline, W. C. Krathwohl, H. N. Laden, J. C. Lapsley, R. A. Leibler, A. L. Lewis, D. R. Lintvedt, N. M. Livesay, M. I. Logs-

don, A. T. Lonseth, Morris Marden, C. W. Mendel, Karl Menger, A. N. Milgram, W. L. Mitchell, M. G. Moore, C. W. Moran, Marston Morse, E. J. Moulton, S. B. Myers, Ivan Niven, E. N. Oberg, J. W. Odle, Rufus Oldenburger, C. D. Olds, Isaac Opatowski, G. H. Peebles, P. M. Pepper, I. E. Perlin, Samuel Perlis, G. B. Price, Tibor Radó, P. V. Reichelderfer, W. T. Reid, Haim Reingold, C. V. Robinson, M. D. P. Rochford, W. H. Roever, A. E. Ross, R. G. Sanger, R. D. Schafer, Peter Scherk, O. F. G. Schilling, K. C. Schraut, G. E. Schweigert, H. A. Simmons, A. H. Smith, W. S. Snyder, N. E. Steenrod, R. C. Stephens, B. M. Stewart, E. B. Stouffer, H. P. Thielman, C. J. Thorne, C. B. Tompkins, P. L. Trump, A. W. Tucker, A. E. Turner, S. M. Ulam, V. J. Varineau, Bernard Vinograde, R. W. Wagner, H. S. Wall, K. W. Wegner, J. V. Wehausen, G. W. Whitehead, L. R. Wilcox, J. E. Wilkins, František Wolf, R. S. Wolfe, Ruth Wolkow, Y. K. Wong, J. W. T. Youngs, Antoni Zygmund.

The meeting opened Friday morning with two sections, analysis with Professor L. M. Graves presiding, and geometry and the theory of numbers with Professor V. G. Grove presiding. Saturday morning there were three sections, analysis with Professor J. V. Atanasoff presiding, algebra, Professor Reinhold Baer presiding, and geometry and topology, Professor N. E. Steenrod presiding. All sectional sessions were held in Eckhart Hall.

On Friday afternoon the Symposium Lecture was given by Professor L. M. Blumenthal with the title *Some imbedding theorems and characterization problems of distance geometry*. Professor Karl Menger presided at this lecture which was given in Rosenwald Hall.

On Friday afternoon following the Symposium Lecture, a tea was given by the ladies of the Department of Mathematics of the University of Chicago.

The attendance at the dinner on Friday evening at the Quadrangle Club was one hundred two. Professor Dunham Jackson served as toastmaster and introduced President Marston Morse who spoke on the present status of deferments for teachers of mathematics and stressed the danger of a serious shortage of qualified teachers at the college level. Following this talk Professor L. W. Cohen and Dean Walter Bartky gave short talks.

At the various sectional meetings on Saturday morning there was read a telegram from a group of Russian mathematicians addressed to all American mathematicians, expressing feelings of scientific unity and a belief in the triumph of our common cause.

There were two meetings of the Council, one being a joint session with the Board of Governors of the Mathematical Association of America. The joint session was held at 12:15 P.M. in the Quadrangle Club and the regular meeting of the Council at 8:45 P.M. in the Common Room of Eckhart Hall at the University of Chicago on April 17, 1942.

At the joint meeting of the Council and Board of Governors President Marston Morse reported on the steps which had been taken concerning the deferment of mathematicians and the procedures recommended to secure proper classification of competent teachers and graduate students by Selective Service Boards. The question of the use of all mathematicians who are inducted into the armed forces was also discussed and President Morse was authorized to make further investigations concerning this problem.

The Secretary announced the election of the following seventy-six persons to membership in the Society:

Mr. Blair Miller Bennett, College of the City of New York;
Dr. Ferdinand Charles Biesele, University of Utah;
Professor Conrad B. Bilgery, Regis College, Denver, Colo.;
Professor Warren Hill Brothers, Jr., Talladega College, Talladega, Ala.;
Mr. Hugh Daniel Brunk, The Rice Institute;
Dr. Arthur Walter Burks, University of Pennsylvania;
Sister Mary Roberdette Burns, Clarke College, Dubuque, Iowa;
Mr. Albert Cahn, Jr., University of Chicago;
Dr. Walter Francis Cassidy, St. John's University, Brooklyn, N. Y.;
Mr. A. John Coleman, University of Toronto;
Miss Clara Marie Cooper, St. Vincent's College, Shreveport, La.;
Miss Jane E. Crawford, American Mathematical Society, New York, N. Y.;
Mr. Anthony B. Crawley, West Virginia State College, Institute, W. Va.;
Professor Ray Harbaugh Dotterer, Pennsylvania State College;
Dr. Ralph Waldo Erickson, Hibbing Junior College, Hibbing, Minn.;
Mr. Edward Allen Fay, University of Rochester;
Mr. Edwin Howard Feather, Valley Forge Military Academy, Wayne, Pa.;
Dr. Hans Fried, Sproul Observatory, Swarthmore College;
Professor Israel Everette Glover, Fayetteville State Teachers College, Fayetteville N. C.;
Professor John M. González, Spring Hill College, Spring Hill, Ala.;
Sister Agnes Ann Green, Immaculate Heart College, Los Angeles, Calif.;
Dr. Frederick William Green, St. Louis Southwestern Railway Lines, St. Louis, Mo.;
Professor William V. Houston, California Institute of Technology;
Mr. Oscar Branche Jackson, Shell Oil Company, Tulsa, Okla.;
Dr. Haym Jaffe, Penn Treaty Junior High School, Philadelphia, Pa.;
Mr. William Katz, New York, N. Y.;
Dr. David Milton Krabill, Potomac State School of West Virginia University, Keyser, W. Va.;
Sister Mary Pachomia Lackay, St. Teresa's College, Kansas City, Mo.;
Miss Jane Marie Lawler, Kansas City, Mo.;
Mr. Herbert R. Leifer, United States Department of Labor, Philadelphia, Pa.;
Mr. Harold Wilbert Linscheid, Bluffton College, Bluffton, Ohio;
Mr. Robert Lorenzen, New York, N. Y.;
Professor John Joseph Lynch, Fordham University, New York, N. Y.;
Dr. Henry Berthold Mann, Columbia University;
Mr. Ernest Percy Miles, Jr., Naval Station, New Orleans, La.;
Professor Carman Edgar Miller, University of New Brunswick;

Mr. Arthur George Montgomery, University of Minnesota;
Dr. Elliott Waters Montroll, Cornell University;
Mr. Joseph Baxter Nelson, University of Southern California;
Sister Mary Cyprian Nightwine, Xavier University, New Orleans, La.;
Mr. Henry Mortimer Nodelman, College of Pharmacy, University of Connecticut;
Dr. Nilan Norris, Hunter College;
Mr. Kenneth Alva Norton, Federal Communications Commission, Washington, D. C.;
Sister Anna Concilio O'Neill, College of Saint Elizabeth, Convent, N. J.;
Mr. Milton Oliver Peach, Carnegie Institute of Technology;
Dean Henry William Pietenpol, Central College, Pella, Iowa;
Mr. George Piranian, The Rice Institute;
Mrs. Marie Helen Polanyi, Bennington College, Bennington, Vt.;
Professor Joseph William Porter, Bluefield State Teachers College, Bluefield, W. Va.;
Miss Ruth Ellen Porter, Albany Senior High School, Albany, Ore.;
Professor Francis McConnell Pulliam, Lambuth College, Jackson, Tenn.;
Mr. Lawrence Lee Rauch, Princeton University;
Dr. Helene Reschovsky, Baldwin School, Bryn Mawr, Pa.;
Professor Charles Kendall Robbins, Purdue University;
Mr. Ira Rosenbaum, Camp Croft, S. C.;
Mr. Joshua Heshel Rosenbloom, Federal Government Service, Philadelphia, Pa.;
Professor Samuel Thomas Sanders, Jr., Southwestern Louisiana Institute, Lafayette, La.;
Mr. William Leete Sandidge, Lynchburg, Va.;
Mr. Milner Baily Schaefer, University of Washington;
Mr. Charles W. Seekins, North Carolina State College;
Mr. Samuel Shulits, United States Engineer Office, Charleston, S. C.;
Mr. Daniel Harold Simmons, University of California at Los Angeles;
Dr. Rolland George Sturm, Aluminum Research Laboratories, New Kensington, Pa.;
Mr. Leonard W. Swanson, University of Minnesota;
Professor Alejandro Terracini, Universidad di Tucuman, Tucuman, Argentina;
Professor Louis F. Tolle, St. John's University, Brooklyn, N. Y.;
Dr. Lewis Fletcher Walton, San Diego State College;
Mr. Alan Wayne, Rhodes School, New York, N. Y.;
Professor Ernst Weber, Polytechnic Institute of Brooklyn;
Mr. Donald Ward Western, Brown University;
Mr. Robert L. White, University of California at Los Angeles;
Mr. Donald Ransom Whitney, Mary Washington College, Fredericksburg, Va.;
Mr. Lloyd Bayard Williams, Georgia School of Technology;
Miss Ruth Wolkow, Barret Junior High School, Louisville, Ky.;
Professor Cecil Benjamin Wright, East Texas State Teachers College, Commerce, Tex.;
Dr. Alexander Wundheiler, College of the City of New York.

It was reported that the following had been elected as nominees on the Institutional Memberships of the institutions indicated:

University of California: Mr. Edward William Barankin, Miss Sarah Margaret Hallam, Messrs. Alfred Horn and Robert Mann.

California Institute of Technology: Messrs. Sherwin Parker Avann, Gerald Harrison, Robert Clarke James, Chia Chiao Lin, Mark Muir Mills, Knox Taylor Millsaps, Wolfgang Kurt Herman Panofsky, Edmund Pinney, William Shand, Jr., Stanley

Cooper Snowdon, Clifford Ambrose Truesdale, and Donald Bingham Wheeler, Jr.
Harvard University: Mr. Robert Wayne Byerly.
New York University: Messrs. Imanuel Marx and Wolfgang Richard Wasow.
Pennsylvania State College: Mr. Ralph P. Bentz.
Wayne University: Mr. Gerald S. Heller.
University of Wisconsin: Messrs. Robert Phillip Eddy, Raymond Scott Embree, John Vernor Finch, Donald R. Lintvedt, and Theodore Edgar Rodd, Miss Marion Dell Wetzel, Mr. Norman Arthur Wiegmann.

Dr. Andrew Paul Guinand of the Royal Canadian Air Force, Chatham, New Brunswick, Canada, was admitted to the Society in accordance with the reciprocity agreement with the London Mathematical Society and Professor Charles Blanc of the University of Lausanne was admitted to the Society in accordance with the reciprocity agreement with the Swiss Mathematical Society.

The following appointments by President Marston Morse were reported: as representative of the Society at the Celebration of the One Hundredth Anniversary of the Founding of Hollins College (Virginia) on May 17–19, 1942, Dr. Nancy Cole; as auditors of the Society's books for 1942, Professors Samuel Borofsky and A. E. Meder, Jr.; as Committee on Arrangements for the Summer Meeting of 1942 at Vassar College, Professors Mary E. Wells (chairman), W. D. Cairns, T. R. Hollcroft, Gertrude Smith, H. S. White; as Committee on Arrangements for the Annual Meeting of 1942 at New York City, Professors G. W. Mullins (chairman), W. D. Cairns, B. P. Gill, T. R. Hollcroft, E. R. Lorch, Mina Rees, J. F. Ritt.

The Secretary reported that the following had been chosen as the voting representatives on the Council of the editorial committees for the year 1942: Bulletin—Dean Tomlinson Fort; Transactions—Professor Einar Hille; Colloquium—Professor M. H. Stone; American Journal of Mathematics—Professor R. L. Wilder; Mathematical Reviews—Professor Oswald Veblen.

An invitation from Oregon State College to hold a meeting of the Society in Corvallis in June, 1943, in connection with the meetings of the Pacific Division of the American Association for the Advancement of Science, was accepted.

The Secretary reported the receipt of a gift of $100 from Professor R. C. Archibald to be used as deemed most desirable by the Librarian.

It was reported by the Bulletin Editorial Committee that, in the future the Index Number of the Bulletin will include indices of abstracts, meetings and papers, but will omit the Annual List of Published Papers and the index to Notes.

The Secretary reported that Professor E. J. McShane had replaced

Professor J. L. Walsh as chairman of the editorial committee for the series, Mathematical Surveys.

Associate Secretary Ingraham reported that the state of the finances of the Society was good, in spite of the present emergency. He also reported that approximately ten persons had taken advantage of the privilege of the reduction in dues offered to enlisted men in the armed forces of the United States and Canada.

Titles and cross references to the abstracts of papers follow below. The papers were read as follows: Papers 1–8 in the section for analysis on Friday morning; papers 9–16 in the section for geometry and the theory of numbers on Friday morning; papers 17–22 in the section for analysis on Saturday morning; papers 23–29 in the section for algebra on Saturday morning; papers 30–36 in the section for geometry and topology on Saturday morning; and papers 37–47, whose abstract numbers are followed by the letter *t*, were read by title. Professor Jonah was introduced by Professor H. K. Hughes, Mr. Tomlinson by Dr. J. J. DeCicco, Mr. Rommel by Professor L. W. Cohen, Dr. Begle by Professor R. L. Wilder, Dr. Swain by Professor S. M. Ulam, and Professor Sadowsky by Professor L. R. Ford. Paper 1 was read by Dr. Bernstein, paper 5 by Professor Morse, paper 11 by Dr. Scherk, paper 13 by Professor Pepper, paper 18 by Dr. Hellinger, paper 20 by Professor Miller, paper 21 by Dr. Hyers, and paper 22 by Professor Cohen.

1. Dorothy L. Bernstein and S. M. Ulam: *On the problem of completely additive measure in classes of sets with a general equivalence relation.* (Abstract 48-5-169.)

2. W. S. Snyder: *Non-parametric surfaces of bounded variation.* (Abstract 48-5-192.)

3. P. V. Reichelderfer: *On bounded variation and absolute continuity for parametric representations of continuous surfaces.* (Abstract 48-5-187.)

4. Tibor Radó: *On a problem of Geöcze.* (Abstract 48-5-186.)

5. Marston Morse and G. M. Ewing: *The non-regular case in the variational theory in large.* (Abstract 48-5-178.)

6. Fritz John: *The character of solutions of linear partial differential equations.* (Abstract 48-5-173.)

7. František Wolf: *On the limits of harmonic and analytic functions along radii which form a set of positive measure.* (Abstract 48-5-199.)

8. William Karush: *A sufficiency theorem for an isoperimetric problem in parametric form with general end conditions.* (Abstract 48-7-236.)

9. H. F. S. Jonah: *Congruences connected with the solution of a certain Diophantine equation.* (Abstract 48-5-155.)

10. R. D. James: *On Euler's conjecture.* (Abstract 48-5-153.)

11. Emil Artin and Peter Scherk: *On the sum of two sets of integers.* (Abstract 48-5-143.)

12. Herbert Busemann: *Spaces with convex spheres.* (Abstract 48-5-206.)

13. B. J. Topel and P. M. Pepper: *Imbedding theorems under weakened hypotheses.* (Abstract 48-5-213.)

14. J. E. Wilkins: *A characterization of the quadric of Wilczynski.* (Abstract 48-5-214.)

15. T. W. Tomlinson: *Geometry of linear fractional polygenic functions.* (Abstract 48-5-211.)

16. A. E. Ross: *On a theorem of Kloosterman.* (Abstract 48-7-232.)

17. W. H. Roever: *Comment on the derivation of the law of perfect gases.* (Abstract 48-5-188.)

18. E. D. Hellinger and H. S. Wall: *Contributions to the analytic theory of continued fractions.* (Abstract 48-5-172.)

19. Isaac Opatowski: *Confluent hypergeometric functions and Markoff chains.* (Abstract 48-5-179.)

20. G. T. Miller and H. K. Hughes: *Analytic continuation of functions defined by factorial series.* (Abstract 48-5-177.)

21. S. M. Ulam and D. H. Hyers: *Approximate isometries of the space of continuous functions.* (Abstract 48-5-194.)

22. J. D. Rommel: *On conservative transformations of functions of two variables.* (Abstract 48-7-244.)

23. Bernard Vinograde: *Split rings and their representation theory.* (Abstract 48-5-167.)

24. Rufus Oldenburger: *Expansions of quadratic forms.* (Abstract 48-5-160.)

25. A. A. Albert: *The radical of a non-associative algebra.* (Abstract 48-5-141.)

26. Ivan Niven: *On matric polynomials.* (Abstract 48-5-159.)

27. R. D. Schafer: *Alternative algebras over an arbitrary field.* (Abstract 48-5-164.)

28. R. E. Johnson: *On structures of infinite modules.* (Abstract 48-5-154.)

29. R. W. Wagner: *Expressibility relations for bilinear operations.* Preliminary report. (Abstract 48-5-168.)

30. C. B. Tompkins: *Local imbedding of Riemannian spaces.* (Abstract 48-5-212.)

31. M. M. Day: *Compactness of a space with monotone closure.* (Abstract 48-5-220.)

32. Samuel Kaplan: *Homologies in metric separable spaces.* (Abstract 48-5-222.)

33. J. W. T. Youngs: *On parametric representations of surfaces.* II. (Abstract 48-5-229.)

34. E. G. Begle (National Research Fellow): *Intersections of absolute retracts.* (Abstract 48-5-219.)

35. R. L. Swain: *Proper and reductive transformations.* (Abstract 48-5-225.)

36. A. N. Milgram: *Dimension of the points with at most n rational coordinates.* (Abstract 48-5-223.)

37. M. S. Webster: *A convergence theorem for certain Lagrange interpolation polynomials.* (Abstract 48-5-197-*t*.)

38. M. A. Sadowsky: *Approximation by rational sequences.* (Abstract 48-5-163-*t*.)

39. H. M. Schwartz: *On some general series expansions.* Preliminary report. (Abstract 48-5-189-*t*.)

40. Walter Leighton and W. J. Thron: *On the convergence of continued fractions.* (Abstract 48-3-112-*t*.)

41. A. R. Schweitzer: *On a class of ordered* $(n+1)$*-ads relevant to the algebra of logic.* I. (Abstract 48-3-134-*t*.)

42. A. R. Schweitzer: *On a class of ordered* $(n+1)$*-ads relevant to the algebra of logic.* II. (Abstract 48-3-135-*t*.)

43. Reinhold Baer: *Radical ideals.* (Abstract 48-5-146-*t*.)

44. George Pólya: *On the combinatory analysis of classifications and permutations.* (Abstract 48-5-184-*t*.)

45. M. L. Kales: *Tauberian theorems.* (Abstract 48-5-174-*t*.)

46. R. F. Clippinger: *Matrix products of matrix powers.* (Abstract 48-5-149-*t*.)

47. R. M. Thrall: *On the decomposition of modular tensors.* (Abstract 48-5-166-*t*.)

W. L. Ayres,
Associate Secretary

ROBERT HENDERSON—IN MEMORIAM

In the death of Robert Henderson on February 16, 1942, American mathematics lost one of its ablest leaders. Those who came into close association with him were deeply impressed not only by his ability but even more by the integrity of his intellect and character; they could not but feel that it was a privilege to know a man of such extraordinary parts.

Mr. Henderson was born in Russell, Ontario, on May 24, 1871. He graduated from the University of Toronto in the Honours School of Mathematics at the head of his class in 1891 and received an appointment as Fellow in Mathematics for the following year. In 1892 the young man entered the Government Insurance Department at Ottawa and by 1896 he had passed the examinations of the Institute of Actuaries of Great Britain and had become a Fellow. Soon thereafter he entered the employ of the Equitable Life Assurance Society of the United States and transferred his residence to this country. Here he progressed rapidly to the top of the profession and became Actuary in 1911. In 1929 he was made Vice President of the company, which position he held until 1936 when he retired.

While nearly all of the two score papers which he published were concerned with actuarial problems, he never lost his interest in pure mathematics. As early as the meeting of October 1895 he presented a paper to the American Mathematical Society which was published in November of the same year. It is entitled *Moral values* and deals with some fundamental relations between probability and insurance. He was an omnivorous reader and found time to keep up with many of the modern developments of mathematics. In the Annals of Mathematics, volume 24, he has a brief paper entitled *Geodesic lines in Riemann space*, which was stimulated by his interest in the theory of relativity.

Mr. Henderson was prominent in the counsels of the Actuarial Society and served a term as Secretary, four years as Vice President, and two years as President. His contributions to the advancement of theory in the actuarial field were varied in character, but his work on interpolation and graduation and on frequency curves and moments is perhaps the best known.

In the early days of the Society, Professor Emory McClintock, who was its president for the period 1891–1894, made a profound impression on the development of actuarial science in America; Mr. Henderson proved to be a worthy successor. He was the recipient of

many honors; for his signal leadership in his field his Alma Mater honored him with the degree of Doctor of Science in 1930. For the Annual Meeting of 1924 in Washington, the Society invited him to give the Josiah Willard Gibbs lecture; his title was *Life insurance as a social service and as a mathematical problem* (see this Bulletin, vol. 31 (1925), pp. 227–252, for the text). In 1927 he was elected to membership in the American Philosophical Society.

When the Society was in sore financial straits in 1924 and Professor J. L. Coolidge was requested to head a movement to raise an endowment fund, Mr. Henderson rendered valiant service; he was in that connection appointed along with G. E. Roosevelt, the banker, and Professor W. B. Fite, the treasurer, on a Committee on Investments. During the succeeding years and until 1940 he served the Society on the financial side giving sound advice regarding investments and other features of the financial problems. During much of that period he served on the Board of Trustees, where his counsel was invaluable. His interest in the Society was further evinced by his gift of one thousand dollars on retirement from the Board of Trustees in 1940. His donation was set up as a separate Fund bearing his name. He was generous also with his time in directions outside his immediate circle; as examples it may be cited that he was consultant to the Church Pension Fund of the Episcopal Church when it was established in 1915, was several times an adviser to the Director of the U. S. Census, and was for years a member of the Trustees of the Teachers Insurance and Annuity Association of America.

R. D. MURPHY
R. G. D. RICHARDSON

BOOK REVIEWS

The Philosophy of Alfred North Whitehead. Edited by Paul Arthur Schilpp. (The Library of Living Philosophers, vol. 3.) Evanston and Chicago, Northwestern University, 1941. 20+745 pp. $4.00.

This book is the third volume of a series entitled *The Library of Living Philosophers* published at Northwestern University under the direction of Professor Paul Schilpp. The basic aim of the series is to effect a clearer understanding of contemporary thinkers by permitting critics to ask them crucial questions which they are to answer. Each volume purports to be the edited record of this journalistic version of the Socratic method. Unfortunately, the present volume does not realize this aim. Professor Whitehead's health did not allow him to answer his critics. Nonetheless, there are interesting essays in the volume, and the publishing of some of Whitehead's latest papers adds to its worth. Especially valuable is the eleven page *Autobiographical notes*, excellent as a revelation of Whitehead, the man. It is difficult to agree with the editor, however, that the essays entitled *Mathematics and the good* and *Immortality* are summaries of Whitehead's philosophy. It is even more difficult to consider them answers to his critics. They should be thought of as late chapters in the Whiteheadian corpus.

But what does this volume contain by way of interest to the mathematician? There is little that will attract the pure mathematician. The only possible exception would be W. V. Quine's essay, *Whitehead and the rise of modern logic*, but even this embraces too many of the evils of secondary sources. There are some interesting points regarding Whitehead's contributions to Boolean algebra in his *Treatise on Universal Algebra*, but these can be appreciated adequately only by a study of the original. By far the largest portion of Quine's essay is devoted to the monotonous task of sketching the high points of *Principia Mathematica*. If one is familiar with mathematical logic this offers little that is new. If one is not familiar with mathematical logic, it is doubtful whether the essay will serve as an adequate introduction to the subject.

Mathematicians with interest in the philosophical aspects of relativity theory may find something to their taste in the essays by F. S. C. Northrop and E. B. McGilvary. These are entitled respectively, *Whitehead's philosophy of science*, and *Space-time, simple location, and prehension*. Those with a general interest in philosophy will profit most from the volume. There are many essays covering a variety of

philosophical themes such as, *Whitehead's theory of value* and *Whitehead's idea of God*.

The book as a whole is well designed, and the print is good. There is an occasional typographical error such as formula (12) on page 150 which should read,

$$[\hat{\alpha} \sim (\alpha \in \alpha) \in \hat{\alpha} \sim (\alpha \in \alpha)] \equiv \sim [\hat{\alpha} \sim (\alpha \in \alpha) \in \hat{\alpha} \sim (\alpha \in \alpha)].$$

A. R. TURQUETTE

Introduction to Logic and to the Methodology of Deductive Sciences. By Alfred Tarski. Enlarged and Revised Edition. New York, Oxford University Press, 1941. 18+239 pp. $2.75.

This is an amplified and revised version of a book which first appeared in Polish in 1936, and was translated into German in 1937. The intention of the original book was to give an elementary but clear account of the concepts of modern mathematical logic for the benefit of readers interested in mathematics but with no technical knowledge of it beyond that possessed by a well trained college freshman. In the English version various additions have been made to make the work more suitable as a textbook for college courses.

This book and its preceding editions have been already reviewed in several places; in particular the German version was reviewed in this Bulletin (vol. 44, p. 317) by Quine. For a considerable list of other reviews see the indexes to volumes 4 (1939) and 6 (1941) of the Journal of Symbolic Logic—on pp. 193 and 187 respectively—; to the lists there given should be added the review by Frink in Mathematical Reviews, vol. 2 (1941), p. 209. In view of this fact it is superfluous for the present reviewer to do more than summarize the general purport of these reviews and to add to the criticisms certain amplifications of his own.

All reviewers, including the present one, are agreed that this is a work of exceptional merit. For the purpose for which it was originally designed it is a masterpiece of exposition. Whether the patching which the book has received to convert it into a textbook will succeed in that endeavor is doubtful—for some there will not be enough technique and for others not enough application to extra-mathematical domains—; but the value of the book for the independent reader is enhanced thereby. The exercises at the ends of the chapters are an excellent feature. For the seasoned mathematician the book is, perhaps, too easy, and it certainly does not give an adequate idea of the difficulty of some logical problems; nevertheless it contains material of interest and value. Within the limitations imposed by its objec-

tives it is the most accurate and perspicuous survey of its subject which now exists.

A defect which has been noticed by several reviewers is an unwarranted departure from accepted terminology. This is especially patent in the case of the algebra of propositions. Here the book avoids the word 'proposition' altogether; in the place of it and its derivatives we find such words as 'sentence,' 'sentential calculus,' and so on. There are two circumstances which probably have a bearing on this choice of terminology. In the first place in the German language there is no distinction analogous to that between the English words 'proposition' and 'sentence'; there are, to be sure, two words, 'Satz' and 'Aussage,' but the distinction between them is not parallel. Now those who prefer the 'sentence' terminology have generally come under the influence of the Vienna Circle, which originated, of course, in a German-speaking community; and it is likely that some of these persons do not sense the violence they are doing to the linguistic instincts of those for whom the English language is native. (It is said that Carnap, since he has lived in America, has become more sympathetic to the use of 'proposition.') In the second place the word 'proposition' seems vague and has for some a metaphysical connotation; these propositions must be mysterious beings indeed, since we have no exact criteria for identity amongst them; moreover they are unnecessary.

In defense of the more usual terminology, reply may be made as follows. In the first place it is purely a mistake in translation to foist upon the English language an incongruous Germanism. Moreover, the notion of sentence is just as vague as that of proposition. Indeed consider the following lines of type (suggested by Church's review—see the list above cited):

(1) $2+3=5$
(2) The sum of 2 and 3 is 5.
(3) The sum of 2 and 3 is 5.
(4) The sum of 2 and 3 is 5.
(5) *The sum of 2 and 3 is 5.*
(6) Die Summe von 2 und 3 ist 5.
(7) Die Summe von 2 und 3 iſt 5.
(8) $3+2=5$
(9) $2+3\neq 5$

How many sentences are there? For Lesniewski presumably there would be $9n$, where n is the number of copies of this magazine; for a typographer, 8; for Quine, who says he regards a sentence as a shape, 7; for a grammarian 5; for a German logician, understanding 'sen-

tence' as translation of 'Satz,' there might be 3. (Carnap's usage in the *Logical Syntax* is distinct from any of the first four; for when Countess Zeppelin translated his book into English she was compelled to translate inside quotation marks in order to preserve the meaning which Carnap wished to convey, which translation would have been improper if any of the first four senses had been intended.) Moreover the use of 'proposition' does not connote a belief in mysterious entities running around the universe; one can simply understand a proposition as a sentence in the above logician's sense. Finally the vagueness inherent in both the words 'proposition' and 'sentence' is irrevelevant since in logic we have no concern with equality of propositions but only their equipollence. In short the replacement of 'proposition' by 'sentence' serves no useful purpose and introduces, rather than eliminates, confusion. This is the opinion of every mathematician, so far as the reviewer knows, who has commented on the subject; Quine, who would doubtless disagree, is a philosopher. It should be noted that there is no departure from accepted usage in the German version; there 'Satz' and 'Aussagenkalkül' are used in their usual senses.

There is a related but distinct criticism in regard to the interpretation of the propositional algebra. Tarski uses the syntactical interpretation, whereby the variables 'p,' 'q' are to be replaced by sentences and the connectives ('$\vee$,' '$\rightarrow$,' '$\sim$') are conjunctions. Another interpretation is possible, namely: where these variables are replaceable by nouns, that is, noun clauses such as 'that 2 precedes 3.' On the first interpretation '$p \rightarrow q$' gives rise to the sentence

if 2 *precedes* 3, *then* 3 *follows* 2;

on the second to the complex clause

that (*that* 2 *precedes* 3) *implies* (*that* 3 *follows* 2),

which, when preceded by the assertion sign (or some similar indication) becomes the sentence

that 2 *precedes* 3 *implies that* 3 *follows* 2.

Either interpretation can be used, the second having the advantage of making the algebra have characteristics more similar to those of mathematical theories in general. As Quine points out, Tarski vacillates between these two.

A final criticism is that the first chapter, on variables, could profitably be completed by considering the types of bound variables that occur in the infinitesimal calculus—for example, in

$$\int_1^2 x^2 dx, \frac{d}{dx} x^2.$$

Such an addition might lead up to an explanation of the Church λ-quantifier, in terms of which, in connection with constants, all other quantifiers can be defined.

HASKELL B. CURRY

Two-Dimensional Potential Problems Connected with Rectilinear Boundaries. By B. R. Seth. Allahabad, India, The Allahabad Law Journal Press, 1939. 118 pp.

This attractive little volume grew out of a course of lectures delivered by Dr. Seth at the University of Lucknow in 1939. It deals primarily with the application of the Schwarz-Christoffel transformation in the solution of several problems and potential theory and related fields of mathematical physics. Among the problems discussed are special cases of the problem of torsion of a long prism as well as the Saint-Venant flexure problem, and problems of ideal fluid flow around prisms.

The author restricts himself to rectilinear boundaries for which the Schwarz-Christoffel transformation allows mapping on a half-plane. By confining himself to rectangular regions, special triangular regions such as the equilateral triangle, the 90°, 60°, 30° triangles and other similar special regions, he is able to express the mapping and the solutions of the problems for them in terms of the classical elliptic functions. Among other regions considered is the "angle-iron," the region on the outside of a rectangle, and the *L*-section.

The book will be welcomed by workers in this field of mathematical physics, as well as by mathematicians who are interested in application of elliptic functions.

H. PORITSKY

Operational Methods in Applied Mathematics. By H. S. Carslaw and J. C. Jaeger. Oxford, Clarendon Press, 1941. 16+264 pp. $5.00.

In May 1815 Simeon Denis Poisson read before the Paris Academy a memoir on the distribution of heat in solid bodies. Extracts from this memoir were at once published in the Journal de Physique and in the Bulletin de la Société Philomathique. The memoir was subsequently enlarged and became, perhaps, one of Poisson's favorites because in May 1821 the work was printed and distributed privately two years before its final publication in Journal de l'École Polytechnique, vol. 12, no. 19, pp. 1–162. Poisson here made, I think, the first use of the method of the inverse Laplace transformation. In an attempt to find the distribution of temperature in a uniform rod radiating at its ends he was led to two linear functional differential equa-

tions for the unknown function f in his general solution of the equation of the conduction of heat. To solve these equations Poisson formed two Laplacian integrals involving the function f, the argument of the exponential function being xz in one case and $-xz$ in the other. By an integration by parts, equations were found from the functional differential equations which led to the values of the integrals. Poisson then deduced the value of a Fourier integral involving the function f and finally derived f by means of the Fourier inversion formula. He gave indeed a formal derivation of the inversion formula for the Laplace integral, a formula which is usually written as an integral along a vertical line in the complex plane.

Poisson thus initiated a method which has been found particularly useful during the last forty years. When V. Pareto in 1892 used the multiplication theorem for Laplacian integrals to solve a type of integral equation and M. Lerch in 1892 proved the uniqueness theorem for the representation of a function by a Laplacian integral involving a continuous function f, the method became a powerful mathematical tool. Further progress was made in 1896 when H. Mellin gave a rigorous discussion of the inversion formula used by Poisson. In 1902 H. M. Macdonald used the inversion formula for the evaluation of some definite integrals and in 1903 Lerch made a similar application of his theorem when his work was republished in Acta Mathematica, volume 27. This volume happened to contain also an account of Fredholm's pioneer work on linear integral equations and so the men interested in integral equations became acquainted also with Lerch's work. Sketches of the method under discussion and its application to differential equations and the evaluation of definite integrals appeared in a Smith's Prize essay of 1905, in a report on integral equations and in a paper of 1910 on radioactive transformations. The subsequent history of the subject and its connection with the operational calculus of Heaviside is given in the historical introduction of the book under review. This book is particularly welcome now that the subject is widely taught because the exposition is good and there are numerous illustrative examples taken from the subjects of dynamics, electric circuits, the conduction of heat, hydrodynamics, electric waves and diffusion. There is also a large collection of examples to enable a student to develop his skill.

The appendices contain a proof of Lerch's theorem, a note on Bessel functions, a discussion of impulsive functions and a treatment of two-point boundary value problems for ordinary linear differential equations. There is also a table of Laplace transforms.

H. BATEMAN

NOTES

A pamphlet entitled *Some military applications of elementary mathematics* has been prepared by the Department of Mathematics at the United States Military Academy at West Point. The material involves mathematics of high school and freshman grade. This pamphlet is being used at the Institute of Military Studies, University of Chicago, and may be obtained by qualified institutions for $0.15 a copy. Address Director, Institute of Military Studies, University of Chicago, Chicago, Illinois.

Professor E. T. Copson of University College, Dundee, and Professor W. H. McCrea of Queen's University, Belfast, have been awarded the Keith Prize of the Royal Society of Edinburgh.

The Chauvenet Prize has been awarded to Associate Professor Saunders MacLane of Harvard University for his two papers *Modular fields* and *Some recent advances in algebra* published in the American Mathematical Monthly.

Professor W. D. Cairns, Secretary-Treasurer of the Mathematical Association of America, has announced that the Department of Mathematics of the University of Toronto has won the first prize of $400 in the fifth annual William Lowell Putnam Mathematical Competition, held March 7, 1942. The members of the team were K. S. Hoyle, H. V. Lyons, M. A. Preston. The second prize of $300 is awarded to the Department of Mathematics of Yale University, the members of whose team were F. H. Brownell, A. M. Gleason, A. E. Roberts. The third prize of $200 is awarded to the Department of Mathematics of the Massachusetts Institute of Technology, the members of whose team were E. D. Calabi, W. S. Loud, G. P. Wachtell. The fourth prize of $100 is awarded to the Department of Mathematics of the College of the City of New York, the members of whose team were Herman Chernoff, Harvey Cohn, Edward Gordon. In addition to these prizes to the departments of mathematics with winning teams, a prize of $50 each is awarded to the following five persons whose scores ranked highest in the six-hour examination (names listed alphabetically): Harvey Cohn, A. M. Gleason, W. S. Loud, H. V. Lyons, M. A. Preston. One of these five will be chosen to receive a $1000 year scholarship at Harvard University, this award to be announced later. The members of the four winning teams will receive individual cash awards according to the ranks of their teams, and all individuals receiving awards will also receive medals. Honorable

mention has been awarded this year to four teams and to five individuals. The teams are from the Department of Mathematics, Cooper Union Institute of Technology, members of the team being Harold Grad, M. S. Klamkin, Kenneth Robinson; the Department of Mathematics, Harvard University, members of the team being R. M. Bloch, L. S. Shapley, J. A. Zilber; the Department of Mathematics, New York University, members of the team being Melvin Lax, Harold Lewis, Henry Shenker; and the Department of Mathematics, Swarthmore College, the team members being N. B. Hannay, W. H. Mills, M. S. Raff. The five individuals receiving honorable mention are E. D. Calabi, Massachusetts Institute of Technology; C. P. Gadsden, Tulane University; K. S. Hoyle, University of Toronto; Melvin Lax, New York University; W. H. Mills, Swarthmore College.

Mr. I. M. Vinogradov has been elected to membership in the American Philosophical Society.

Professor A. N. Whitehead has been elected president of the British Mathematical Society.

Mr. Philip Hall and Mr. A. H. Wilson of Cambridge University and Mr. G. H. Henderson of Dalhousie University have been elected fellows of the Royal Society

Dr. Vannevar Bush, president of the Carnegie Institute of Technology, received an honorary doctorate from Rutgers University.

Professor F. A. Foraker has received an honorary doctorate from the University of Pittsburgh.

Professor T. R. Hollcroft of Wells College has received an honorary doctorate from Hanover College.

Professor F. D. Murnaghan of Johns Hopkins University has been elected to membership in the National Academy of Sciences and the American Philosophical Society.

Dr. J. A. Sharpe, a member of this Society, has been elected by the Society of Exploration Geophysicists to a three year term as editor of Geophysics, a quarterly journal of general and applied geophysics.

Dean H. M. Westergaard of Harvard University has been elected to membership in the American Philosophical Society.

Dr. Howard Levi of Columbia University and Dr. L. I. Wade of Duke University have received fellowship appointments for 1942–1943 from the National Research Council. Dr. Levi and Dr. Wade will work at the Institute for Advanced Study.

Guggenheim Memorial Fellowships for work in mathematics have been awarded to Dr. J. C. C. McKinsey and Dr. Alfred Tarski.

Professor Willy Prager of Brown University has requested the correction of information about him which was received by the American Mathematical Society and published on page 25 of volume 48 of this Bulletin. He was acting director of the Institute of Applied Mechanics at Göttingen and professor of mechanics at Istanbul before coming to the United States.

Assistant Professor Harriet W. Allen of Hollins College, Hollins College, Virginia, is temporarily serving as physicist, in the Bureau of Ordnance, Navy Department, Washington, D. C.

Dr. H. C. Ayres of the United States Naval Academy has been promoted to an assistant professorship.

Mr. V. P. Barta has accepted an appointment as research physicist at the Massachusetts Institute of Technology.

Dr. Walter Bartky of the University of Chicago has been promoted to a professorship of applied mathematics.

Assistant Professor Julia W. Bower of Connecticut College has been promoted to an associate professorship.

Associate Professor R. V. Churchill of the University of Michigan has been promoted to a professorship.

Mr. E. C. Coker of Clemson College, Clemson, South Carolina, has been promoted to an assistant professorship.

Assistant Professor Marguerite D. Darkow of Hunter College has been promoted to an associate professorship.

Assistant Professor Paul Eberhart of Washburn Municipal University, Topeka, Kansas, has been promoted to an associate professorship.

Mr. B. G. Farley of Massachusetts Institute of Technology will be a research associate in electrical engineering at the Radiation Laboratory.

Professor W. B. Fite of Columbia University has retired. Professor J. F. Ritt will succeed him as chairman of the department of mathematics.

Assistant Professor J. S. Frame of Brown University has been appointed to an associate professorship at Allegheny College.

Assistant Professor E. E. Haskins of Northeastern University has been promoted to an associate professorship.

Associate Professor C. T. Hazard of Purdue University has been promoted to a professorship.

Assistant Professor E. A. Hazlewood of Texas Technological College has been promoted to an associate professorship.

Professor W. V. Houston of California Institute of Technology is working as a physicist in the Division of National Defense of Columbia University.

Mr. J. F. Kenney has been appointed to an assistant professorship at the University of Wisconsin.

Associate Professor Marie Litzinger of Mt. Holyoke College has been promoted to a professorship.

Assistant Professor Sidney McCuskey of Case School of Applied Science has been promoted to an associate professorship.

Assistant Professor H. L. Miller of the University of Cincinnati has been promoted to an associate professorship.

Associate Professor Deane Montgomery of Smith College has been promoted to a professorship.

Assistant Professor C. W. Munshower of Colgate University has been promoted to an associate professorship.

Dr. D. C. Murdoch of the University of Saskatchewan has been promoted to an assistant professorship.

Associate Professor C. O. Oakley of Haverford College has been promoted to a professorship.

Dr. George Polya has been appointed to an associate professorship at Stanford University.

Assistant Professor E. J. Purcell of the University of Arizona has been promoted to an associate professorship.

Dr. Eric Reissner of Massachusetts Institute of Technology has been promoted to an assistant professorship.

Professor F. G. Reynolds of the College of the City of New York has retired.

Assistant Professor H. J. Riblet of Hofstra College will be re-

search associate at the Radiation Laboratory, Massachusetts Institute of Technology.

Professor P. R. Rider of Washington University is to be an exchange professor at the National University of Mexico under the auspices of the Division of Cultural Relations, United States Department of State.

Assistant Professor Robin Robinson of Dartmouth College has been promoted to a professorship.

Assistant Professor E. A. Saibel of Carnegie Institute of Technology has been promoted to an associate professorship.

Professor S. T. Sanders of Louisiana State University has retired. Professor W. V. Parker will succeed him as head of the department of mathematics.

Dr. L. W. Tordella of Loyola University, Chicago, Illinois, has been promoted to an assistant professorship.

Dr. Abraham Wald of Columbia University has been promoted to an assistant professorship.

Dr. J. F. Wardwell of Colgate University has been promoted to an assistant professorship.

Dr. Antoni Zygmund of Mt. Holyoke College has been promoted to an associate professorship.

Dr. P. R. Halmos has been appointed associate at the University of Illinois.

Mr. Thorstein Larsen has been appointed associate in electrical engineering at Johns Hopkins University.

Dr. Abe Gelbart of North Carolina State College, Raleigh, will be on leave for the academic year 1942–1943 at Brown University.

The following appointments to instructorships are announced: Air Corps Technical School, Scott Field, Illinois: Dr. Louis Garfin; Amherst College: Mr. N. D. Nelson; University of California (Berkeley): Dr. František Wolf; University of Colorado: Mr. George Ulrich; Columbia University: Miss Margaret P. Martin; Compton Junior College: Dr. A. E. Marston; Connecticut College: Miss Alice E. Turner; Cornell University: Dr. C. E. Rhodes; Harvard University: Dr. Harry Pollard; University of Illinois: Dr. R. W. Hamming; Johns Hopkins University: Dr. G. W. Whaples; Louisiana State

University: Dr. K. L. Nielsen; Loyola University, New Orleans, Louisiana: Mr. E. H. Larguier; Montana State University: Mr. Roy Dubisch; Naval Pre-Flight Training School, Chapel Hill, North Carolina: Dr. R. P. Boas; Princeton University: Mr. C. A. Truesdell; R.C.A. Institutes, Inc., New York City: Mr. H. M. Nodelman; Rutgers University: Mr. R. B. Kleinschmidt; Stanford University: Dr. J. G. Herriot; United States Merchant Marine Academy: Mr. B. M. Ingersoll; United States Naval Academy: Dr. H. T. Muhly; University of Wisconsin: Dr. R. H. Bruck, Dr. Bernard Vinograde.

Professor Emeritus Susan R. Benedict of Smith College died on April 8, 1942, at the age of sixty-eight years. She had been a member of the Society since 1912.

Professor Emeritus L. S. Hulburt of Johns Hopkins University died on March 29, 1942, at the age of eighty-four years. He had been a member of the Society for thirty-five years.

Professor Byron Ingold of Culver-Stockton College, Canton, Missouri, died March 23, 1942.

Assistant Professor Elizabeth E. Knight of Milwaukee State Teachers College died on March 26, 1942. She had been a member of the Society since 1927.

Professor W. W. Landis of Dickinson College died April 9, 1942, at the age of seventy-three years. He had been a member of the Society since 1897.

Associate Professor Roy MacKay of New Mexico State College died May 12, 1942.

Professor Maria M. Roberts of Iowa State College died on April 12, 1942, at the age of seventy-four years. She had been a member of the Society for twenty-five years.

Professor J. H. Weaver of Ohio State University died April 7, 1942. He had been a member of the Society since 1916.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

230. Reinhold Baer: *Rings with duals.*

The ring R' is said to be a right-dual of the ring R, if there exists a duality between the partially ordered set of all the right-ideals in R and the partially ordered set of all the right-ideals in R'. If the ring R possesses a right-identity element, and if there exists to every quotient-ring of R a right-dual possessing a left-identity element, then both the maximum and the minimum condition are satisfied by the right-ideals in R, and R meets a generalized uni-seriality requirement. A complete theory of the primary rings with right-duals may be developed, determining their structure and their dualities. (Received May 13, 1942.)

231. Nathan Jacobson: *Classes of restricted Lie algebras of characteristic p.* II.

Let $\mathfrak{A}=\Phi(x_1, \cdots, x_m)$ be an associative commutative algebra having the basis $x_1^{k_1}x_2^{k_2}\cdots x_m^{k_m}$, $0\leqq k_i\leqq p-1$ such that $x_i^p=\xi_i$ is in the field Φ of characteristic p. The restricted Lie algebras considered in this paper are the derivation algebras θ of $\mathfrak{A}$'s of the above type. The case where $\mathfrak{A}$ is a field was defined in a previous paper of the author and the case where $m=1$ and $\xi_1=1$ has been considered by Witt, by Zassenhaus and by Ho-Jui Chang. The author proves that θ is normal simple except when $p=2$ and $m=1$. The derivations of θ are all inner and if $p\geqq 5$ the automorphisms of θ have the form $D\to S^{-1}DS$ where S is an automorphism of $\mathfrak{A}$. This implies that if $p\geqq 5$, two algebras $\theta(\mathfrak{A}_1)$ and $\theta(\mathfrak{A}_2)$ are isomorphic if and only if the associative algebras $\mathfrak{A}_1$ and $\mathfrak{A}_2$ are isomorphic. (Received April 3, 1942.)

232. A. E. Ross: *On a theorem of Kloosterman.*

Kloosterman (Acta Mathematica, vol. 49 (1926)) determined conditions under which the form (1) $f=ax^2+by^2+cz^2+du^2$ would represent all large integers and pointed out that there was a finite (and actually small) number of such forms f for which his method failed to yield conclusive results. Upon close examination of these exceptional cases one notices that should one replace the original form f by a suitably chosen equivalent form $f_1=x'Ax$ (this Bulletin, abstract 45-9-369) then one can reduce the problem to the study of a related form of type (1) to which Kloosterman's sufficient conditions apply. Although the need for choosing the quotients of the upper left-hand corner principal minors of A by the order invariants as primes introduces an element of tentativeness, the actual computation is carried out without difficulty. (Received April 17, 1942.)

233. H. Schwerdtfeger: *A complete parametrization of the symplectic group.*

While all known parametrizations of the symplectic group omit certain "exceptional" elements, the parametrization derived in the present paper covers the whole group. Let ϵ be the n-rowed unit matrix and F the matrix with first row 0,0 and second row ϵ,0. A $2n$-rowed real regular matrix T is proved to be symplectic if and only if $T'FT-F=H$ is symmetric (contact condition), and satisfies the condition: $(F'-F)\cdot(H+F)$ is idempotent. To prove the sufficiency of the latter condition one has to show that for any such matrix H a symplectic T can be found with which it is associated by the said relation. By establishing a set of normal forms for H under the transformation $S'HS$ where S is a $2n$-rowed matrix for which $S'FS=F$, that is, S is the matrix with first row σ,0 and second row $0,\sigma'^{-1}$, a set of normal matrices T can be found such that RTS is the most general symplectic matrix where R is the matrix with first row ρ,0; second row $0,\rho'^{-1}$ and ρ,σ are any regular n-rowed matrices. The method has been carried through in detail for $n=2$. (Received May 1, 1942.)

Analysis

234. Einar Hille: *Notes on linear transformations.* IV. *Representation of semi-groups.*

Let $\{T_s\}$ be a semi-group of linear bounded transformations on a separable Banach space to itself, defined for $s>0$. Let T_s be weakly measurable and $\|T_s\|\leq 1$ for $s>0$. Let $T_s(E)$ be dense in E. Put $A_h=(1/h)[T_h-I]$. For $h\to 0$, $T_hx\to x$ everywhere in E and $A_hx\to Ax$ in a dense set $D(A)$. Here A is linear, closed and ordinarily unbounded. The resolvent $R(\lambda)$ of A is the negative of the Laplace transform of T_s and is bounded for $\Re(\lambda)>0$. Conversely, T_s is expressible in terms of $R(\lambda)$ by the inversion formula for Laplace integrals which gives an interpretation of T_s as $\exp(sA)$. A further interpretation is given by $T_sx=\lim_{h\to 0}\exp[sA_h]x$, valid in $D(A)$. The method is essentially that of Stone. (Received April 6, 1942.)

235. Witold Hurewicz: *An ergodic theorem without invariant measure.*

Let E be an abstract space carrying a completely additive measure μ defined on a completely additive field Ω of subsets of E (it is assumed that a set $X\in\Omega$ with $\mu(X)=\infty$ can always be split into a countable number of sets with finite measures). Let T be a one-to-one point transformation of E on itself satisfying the conditions: (1) $X\in\Omega$ implies $T(X)\in\Omega$; (2) $T(X)\subset X\in\Omega$ implies $\mu(X-T(X))=0$ (incompressibility condition). Finally let $F(X)$ $(X\in\Omega)$ be an additive finite-valued set function, absolutely continuous with respect to the measure μ. For $X\in\Omega$, let $F_n(X)=\sum_{i=0}^{n}F(T^i(X))$, $\mu_n(X)=\sum_{i=0}^{n}\mu(T^i(X))$ $(n=0,1,2,\cdots)$. Since F_n is absolutely continuous with respect to the measure μ_n, $F_n(X)=\int_X f_n d\mu_n$, where f_n is a measurable point function defined on E. It can be shown that the sequence $\{f_n\}$ converges almost everywhere to a function ϕ, invariant with respect to T. By "almost everywhere" is meant that the points of divergence form a set M such that $\mu(T^n(M))=0$ for $n=1,2,\cdots$. If the measure μ is finite and invariant with respect to T, this theorem coincides with Birkhoff's ergodic theorem. (Received April 24, 1942.)

236. William Karush: *A sufficiency theorem for an isoperimetric problem in parametric form with general end conditions.*

The problem studied is that of minimizing an integral $I(C)=g(a)+\int_{t_1}^{t_2}f(a,y,y')dt$

in a class of admissible arcs C: a_h, $y_i(t)$ $(h=1, 2, \cdots, r; i=1, 2, \cdots, n; t_1 \leqq t \leqq t_2)$ in ay-space satisfying conditions of the form $y_i(t_s)=y_{is}(a)$ and $I_\sigma(C)=g_\sigma+\int_{t_1}^{t_2} f_\sigma dt=0$ $(s=1, 2; \sigma=1, 2, \cdots, m)$. The parameters a_h are independent of t. It is shown that a set of sufficient conditions for a proper strong relative minimum on an arc E which does not intersect itself is that E satisfy the multiplier rule including the transversality conditions, nonsingularity, the strengthened Weierstrass condition, and that the second variation of I along E be positive. The method of proof is a generalization of methods used by M. R. Hestenes for non-parametric problems. In particular, use is made of a generalized field theory and of properties of broken extremals. Among other things it is shown that in the sufficiency proof one can assume, without loss of generality, that E is an extremal of no integral of the form $c_\sigma I_\sigma$ where the constants c_σ are not all zero, that is, that E is normal relative to the fixed end point case. (Received April 17, 1942.)

237. Walter Leighton and W. J. Thron: *On the convergence of continued fractions to meromorphic functions.*

In the continued fraction $1+K(a_n x/1)$ let the numbers a_n be complex and denote by L (the point ∞ may be in this set) the set of the limit points of the sequence $\{a_{2n+1}\}$. If $\lim a_{2n}=0$, the continued fraction converges to a meromorphic function of the complex variable x in every region contained in the set D, where D is defined as follows: $x \in D$ if $|x+1/k|>\epsilon$ for all $k \in L$, $k \neq 0$, and if $|x|<M$. The constants ϵ and M are arbitrary. In this theorem the role of the sequences $\{a_{2n}\}$ and $\{a_{2n+1}\}$ can be interchanged. (Received May 28, 1942.)

238. A. N. Lowan and H. E. Salzer: *Coefficients in formulae for numerical integration.*

The values of $B_n^{(n)}(1)/n!$ and $B_n^{(n)}/n!$, where $B_n^{(n)}(1)$ denotes the nth Bernoulli polynomial of the nth order for $x=1$ and $B_n^{(n)}$ denotes the nth Bernoulli number of the nth order, were computed for $n=1, 2, \cdots, 20$. The quantities $B_n^{(n)}(1)/n!$ are required in the Laplace formula of numerical integration employing forward differences, as well as in the Gregory formula and the central difference formulae. The quantities $B_n^{(n)}/n!$ are used in the Laplace formula employing backward differences. (See Milne-Thomson, *Calculus of Finite Differences*, pp. 181–187, 191–193.) The above results were obtained in the course of work carried out by Mathematical Tables Project (Work Projects Administration, New York City), on computational tools for numerical integration. (Received May 21, 1942.)

239. Szolem Mandelbrojt and F. E. Ulrich: *On a generalization of the problem of quasi-analyticity.*

If $\{M_n\}$ is a sequence of positive numbers, let $C(M_n)$ be the class of functions infinitely differentiable on $0 \leqq x < \infty$ such that to each function $f(x)$ of the class there corresponds a positive constant k with the property that $|f^{(n)}(x)| < k^n M_n$ $(n \geqq 1)$ for all $0 \leqq x < \infty$. Let (1) $T(r)=\text{l.u.b.}_n(r^n/M_n)$ for $n \geqq 1$. It is known that if a function $f(x)$ belongs to a class $C(M_n)$ such that (2) $\int_1^\infty \log T(r)dr/r^2=\infty$, and if $f^{(n)}(0)=0$ $(n \geqq 0)$, then $f(x) \equiv 0$. In the present paper conditions are obtained under which a function belonging to $C(M_n)$ will be identically zero when it is supposed that only a partial subsequence of its derivatives are zero at $x=0$. For instance, suppose that $f(x)$ belongs to $C(M_n)$, (3) $\int_0^\infty |f(x)|dx < \infty$ and (4) $f^{(\nu_n)}(0)=0$ $(n \geqq 1)$ with (5) $G=\lim \sup_{n\to\infty}(\nu_n/n)<2$. If (6) $\lim \sup_{n\to\infty}|f^{(n)}(0)|^{1/n}<\infty$ and if (2) holds, then $f(x) \equiv 0$. Condition (6) can be re-

moved and the conclusion will still be valid if in (2) $T(r)$ is replaced by $T_\omega(r) =$ l.u.b. $(r^{\omega\lambda_n}/M_{\lambda_n+1})$ for $n \geqq 1$, where $\{\lambda_n\}$ is the sequence of positive integers complementary to the sequence $\{\nu_n\}$ with respect to the non-negative integers, and ω is a positive constant less than $(2/G)-1$. If $f(x)$ is even and periodic, condition (3) can be removed. (Received May 11, 1942.)

240. Marston Morse: *Unstable minimal surfaces bounded by a rectifiable contour.*

Recent theorems of Morse and Tompkins and of Shiffman establish the existence of an unstable minimal surface of disc type under appropriate hypothesis on the minimizing sets of the Douglas Dirichlet integral, and on the bounding curve g. A similar theorem is now proved by the author supposing g is no more than simple and rectifiable, and that at least two relative disjoint minimizing sets belong to the area functional A. The surfaces admitted are harmonic and of disc type. The method of the author does not require the analysis of the harmonic surfaces bounded by polygons as does the method of Courant and Shiffman (announced orally recently but not yet published). The theorem is a corollary of theorems proved by Morse and Tompkins in their recent paper in the Annals of Mathematics on *Minimal surfaces of non-minimum type by a new mode of approximation*. The theorem of Morse and Tompkins on the relative continuity of A covers the most difficult point in the analysis. (Received April 17, 1942.)

241. M. E. Munroe: *On the finite dimensionality of certain Banach spaces.*

It is shown that if B is a Banach space, a necessary and sufficient condition that every unconditionally convergent series of elements of B be absolutely convergent is that B be finite-dimensional. On the basis of this result it is shown that finite dimensionality of the range space is necessary and sufficient that (1) all additive absolutely continuous functions of measurable sets be of bounded variation, (2) all Pettis integrals be of bounded variation, (3) all additive absolutely continuous functions of measurable sets be Bochner-Dunford integrals, and (4) the Pettis and Bochner-Dunford integrals be equivalent. (Received May 13, 1942.)

242. Mary De Pazzi Rochford: *Completely non-integrable pfaffians.*

The pfaffian $\sum P_i dx_i$ is called completely non-integrable if there does not exist even a continuous function λ which is an integrating factor, that is, makes $\int \sum P_i dx_i$ independent of the path. If (1) $P_i(x_1, \cdots, x_n)$ is continuous for $i = 1, \cdots, n-1$, (2) $P_n = 0$, (3) P_1 actually depends upon x_n, that is, $P_1(x_1, \cdots, x_{n-1}, x_n') \neq P_1(x_1, \cdots, x_{n-1}, x_n'')$ for a system $x_1, \cdots, x_{n-1}$, (4) $P_2, \cdots, P_{n-1}$ do not depend upon x_n, then $P_1 dx_1 + \cdots + P_{n-1} dx_{n-1}$ is completely non-integrable. (Received May 20, 1942.)

243. Mary De Pazzi Rochford: *Differentiability properties of certain functions of two variables.*

Let P and Q be two continuous functions of x, y. If $\int P dx + Q dy$ is independent of the path, then for each y_0 and y_1 there exists a function $\eta(x, y_0, y_1)$ such that $Q(x, \eta(x, y_0, y_1))$ has a total derivative with respect to x which is equal to the difference quotient $[P(x, y_0) - P(x, y_1)]/(y_0 - y_1)$. For each continuous function g of one variable, even if g is nowhere differentiable, there exists a function $\eta(x, y_0, y_1)$ such that $g(x + \eta(x, y_0, y_1))$ has a derivative with respect to x which is equal to $[g(x+y_0) - g(x$

$+y_1)]/(y_0-y_1)$. If $\partial(\int Q dy)/\partial x$ exists, then each integrating factor of $Pdx+Qdy=0$ has a similar differentiability property. (Received May 20, 1942.)

244. J. D. Rommel: *On conservative transformations of functions of two variables.*

Necessary and sufficient conditions are given in order that $y(s, t)=\int_0^\infty\int_0^\infty k(s, t, \sigma, \tau)$ $x(\sigma, \tau)d\sigma d\tau$ shall transform $x(\sigma, \tau)$ such that $\lim_{\sigma,\tau\to\infty}x(\sigma, \tau)$ exists into $y(s, t)$ such that $\lim_{s,t}y(s, t)$ exists. From this result a characterization of regularity is obtained. By considering step-kernels and step-functions, regular transformations of double sequences are characterized. (Received April 18, 1942.)

245. Raphael Salem and D. C. Spencer: *The influence of gaps on density of integers*

An infinite sequence of integers is said to have the complete gap property with respect to $\omega(x)$ ($\omega(x)$ being a positive non-decreasing function for $x\geqq 0$) if in every closed interval $(a, a+l)$ ($a\geqq 0$, $l\geqq l_0$) there exists an open interval of length not less than $\omega(l)$ which contains no point of the sequence. Such a sequence will be denoted by $S[\omega(x)]$. Let $\nu(m)$ be the number of terms of the sequence not greater than m. The following theorems are proved: (1) If the integral $\int_0^\infty(\omega(x)/x^2)dx$ diverges, $\lim \nu(m)/m=0$. (2) If $\omega(x)$ is such that $\omega(x)/x^2$ decreases and the integral converges, there exists an $S[\omega(x)]$ with $\liminf \nu(m)/m>0$. (3) If $\omega(x)=\theta x$ $(0>\theta>1/3)$, then $\nu(m)/m=O(m^{-\alpha})$ where $\alpha=[\log(1-2\theta)/(1-3\theta)]/[\log 2(1-2\theta)/(1-3\theta)]$. (4) If $0<\theta<1/3$, there exists a sequence $S[\theta x]$ with $m^{-\alpha}=O(\nu(m)/m)$. (5) If $\omega(x)=\theta x$ $(1/3\leqq\theta<1/2)$, then $\nu(m)=O(\log m)$. (6) If $1/3\leqq\theta<1/2$, there exists a sequence $S[\theta x]$ with $\log m=O(\nu(m))$. (7) If $\omega(x)\geqq x/2$, there exists no infinite sequence $S[\omega(x)]$. (Received April 30, 1942.)

Applied Mathematics

246. H. K. Brown: *The resolution of boundary value problems by means of the finite Fourier transformation.*

The finite sine transformation and the finite cosine transformation are defined as the linear functional operations $\mathfrak{S}\{G\}=\int_0^\pi G(x)\sin nx dx=g_s(n)$ and $\mathfrak{C}\{G\}=\int_0^\pi G(x)$ $\cdot\cos nx dx=g_c(n)$, respectively. The inversion of the product of the transforms of G and H can be made by means of four Faltung theorems. The finite sine transformation was applied to a problem in general heat flow in one dimension in which the nonhomogeneous linear partial differential equation has coefficients which may be functions of the time. It was proved in detail that this problem can be resolved into a standard heat flow problem which has a differential equation of simpler type and constant coefficients. The Faltung theorems permitted an inversion in closed form of the solution of the transformed boundary value problem. A general heat flow problem of similar type in three dimensions was resolved into the same standard problem in one dimension. By the introduction of a fundamental set of solutions of the transformed problem it was possible to make a resolution of the boundary value problems of a general vibrating string, membrane and beam. (Received April 11, 1942.)

Geometry

247. C. J. Everett: *Affine geometry of vector spaces over rings.*

Methods of E. Artin's *Coordinates in affine geometry* (Publication of the University of Notre Dame, Reports of a Mathematical Colloquium, (2), Issue 2) are used to show

that a "plane geometry" subject to a weaker set of axioms on "points" and "lines" permits a point coordinatization $P=(x,y)$ with x, y in a ring R with unit, without zero-divisors, and with every two elements $(a,b)\neq(0,0)$ having a g.c.r.d. $=d$ (d right divisor of a, b divisible by every right divisor of a,b); set of all points on a line: $(x+ak, y+bk)$, x y fixed; a, b fixed, not 0, 0, with g.c.r.d. 1; k arbitrary in R. The intuitive ideas in the "geometry" of the lattice points (integer coordinates) of the affine real plane are thus abstracted in equivalent geometric and algebraic forms. (Received May 20, 1942.)

Statistics and Probability

248. Henry Scheffé: *On the theory of testing composite hypotheses with one constraint.*

Suppose that one has a sample from a population whose distribution function depends on l parameters $\theta_1, \theta_2, \cdots, \theta_l$, and that on the evidence of the sample it is wished to test the hypothesis $H_0:\theta_1=\theta_1^0$. Not only are many statistical problems of practical interest precisely of this type, but the theory of testing such hypotheses with one constraint is intimately related to Neyman's theory of confidence intervals (Proceedings of the Royal Society, 1937). Type B regions for testing H_0 were defined by Neyman (Bulletin Société Mathématique de France, vol. 64, (1935)) who gave a method of finding such regions in the case $l=2$. His theorem is generalized to the case $l\geqq 2$ in the present paper. Type B_1 regions are defined as the natural generalization of the type A_1 regions of Neyman and Pearson (Statistical Research Memoirs, 1936) to permit the presence of the nuisance parameters $\theta_2, \cdots, \theta_l$ in the distribution. Sufficient conditions are given that a type B region be also of type B_1. The paper will appear in the Annals of Mathematical Statistics. (Received May 29, 1942.)

Topology

249. W. H. Gottschalk: *On k-to-1 mappings.*

Let A and B be compact metric spaces and let the mapping $T(A)=B$ be continuous and at most k-to-1, where k is an integer. Let $o(x)$ denote the Urysohn-Menger order of the point x. It is shown that if $y\in B$ has exactly n inverse points $x_1, \cdots, x_n$, then $\sum_{i=1}^{n} o(x_i)\leqq k\cdot o(y)$. Suppose A and B are continua and T is (exactly) k-to-1. It follows from this inequality that if y is an end point of B, then each inverse point of y is an end point of A. Let P denote the property of being a subcontinuum of A on which T is k-to-1. It is proved that if A has (the inducible property) P irreducibly, then B contains no end point; and, in case $k=2$, B has the stronger property of containing no cut point. It follows that no dendrite is a continuous k-to-1, $k>1$, image of a continuum; or, equivalently, every continuous k-to-1, $k>1$, image of a continuum contains a non-cut non-end point. (Received April 11, 1942.)

250. P. R. Halmos and Hans Samelson: *On monothetic groups.*

A topological group is *monothetic* (van Dantzig) if it contains an element (called a *generator*) whose powers are dense in the group. (I) A compact abelian group is monothetic if and only if its character group is algebraically isomorphic to a subgroup of the circle. (II) A compact separable abelian group is monothetic if and only if the quotient group with respect to the component of the identity is such. (Here separability means the existence of a countable dense set.) In particular a compact separable and connected abelian group is always monothetic. (III) Almost every element (Haar

measure) of a compact abelian connected group, satisfying the second countability axiom, is a generator (Schreier and Ulam). The countability restriction is essential in (III); the set of generators of the toral group of dimension continuum has inner measure 0 and outer measure 1. Similarly, without the hypothesis of connectedness the set of generators can have any measure between 0 and 1. It is an easy application of (I) to describe completely the structure of totally disconnected monothetic groups. (Received April 6, 1942.)

251. A. D. Wallace: *Separation spaces.* II.

A definition of compactness is given for a separation space. The use of closure is avoided. It is shown that a compact connected nondegenerate separation space contains at least two non-cut points. For metric spaces this result is well known. This note will appear in Annaes da Academia Brasileira de Sciencias. (Received May 21, 1942.)

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